KWIK CIRCUIT FAQ

Design of an anti-aliasing Sallen-Key filter

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FAQ: How to design an anti-aliasing architecture using Sallen-Key filters.

Introduction

This KWIK (Know-how With Integrated Knowledge) Circuit application note offers a step-by-step guide to address a specific design challenge. For a given set of application circuit requirements, it illustrates how these are addressed using generic formulae and makes them easily scalable to other similar application specifications.

In any sampling system, such as a measurement system involving an ADC, a phenomenon called aliasing can cause the signal at higher frequency band to 'fold down' into the Nyquist frequency band, making them indistinguishable from the signals of interest. The Nyquist frequency being half of the sample rate f_s. The bandwidth of a circuit being sampled by an ADC should be less than half of the sample rate. Aliasing can impact measurement accuracy by contaminating the output with unwanted signals and noise. In **Figure 1** and **Figure 2**, examples of proper sampling and non-proper sampling are shown with high and low sampling rate.

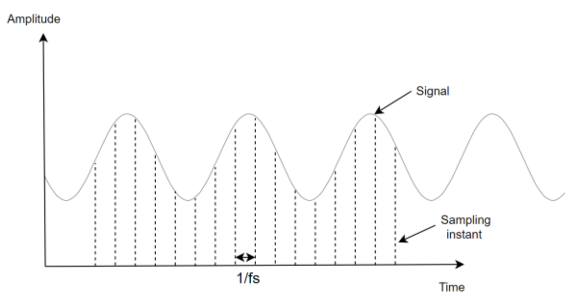

Figure 1: High enough sampling rate to resolve signal.

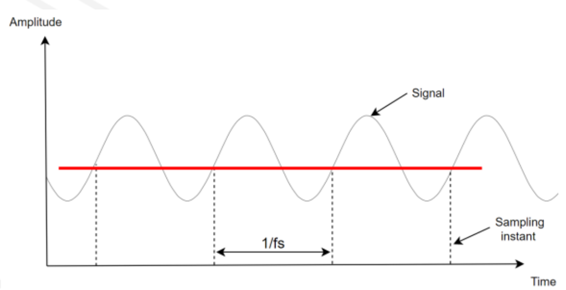

Figure 2: Low sampling rate, aliasing, back to DC.

It is a particular problem for measurement systems used in applications that rely on data analysis in the frequency domain, such as condition-based monitoring, predictive maintenance, power quality monitoring and passive SONAR. These systems typically support wide bandwidth data acquisition and the ability to process signals such as vibration, power and acoustics in the frequency domain, and make decisions based on the profile of the tones and harmonics. The nature of the monitoring requirement makes them particularly sensitive to spectral interferences. Low pass filters are an essential part of signal chain design in these measurement systems. They help prevent the out-of-band signals and noise from folding into the signal bandwidth of interest.

Active filters are usually chosen over passive filters in signal processing circuits since they provide power gain at the output and frequency ranges which can easily be adjusted by modifying the parameters of the filter. The four most common active filters are Butterworth, Elliptical, Bessel and Chebyshev. To choose the most appropriate filter, it really depends on the application and the element that needs to be traded during the filter design process, such as the frequency response, the number of stages or even the phase linearity. **Figure 3** shows the response of each type of active filter and are described further in the design tips section.

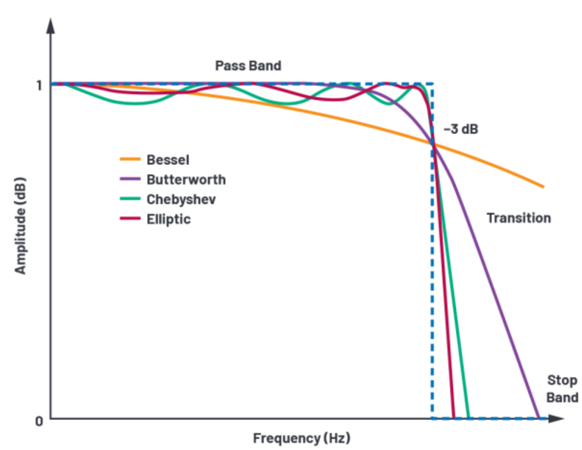

Figure 3: The four types of active filters.

The choice here, will be to implement a Butterworth filter. Once the filter is chosen, the configurations usually being used to support active pole-pair topologies are the Sallen-key filter and the Multiple Feedback filter. This KWIK circuit note will assist with the design and implementation of a low pass Sallen-Key filter to prevent aliasing.

A single stage Sallen-Key filter consists of an active component (an Op-Amp) with passive components (resistors and capacitors). The voltage gain of the Op-Amp is set by a two-resistor voltage divider, and the Op-Amp is used to reject out-of-bounds signals. An example filter circuit design, targeting low power applications is discussed in this article. The principles discussed can be applied for filter design in general. The design will feature the LTC2066, a 10µA supply current, low I_B, Zero-Drift Operational Amplifier (Op-Amp). There are other Op-Amps in our low-power signal chain portfolio which could provide similar results (AD8505; LTC6258; LTC2063; ADA4505; MAX40023; MAX40108).

The circuit in **Figure 4** provides the overall schematic where the LTC2066 is chosen to realize the Sallen-Key, two pole, low pass filter, in unity gain.

Design Specifications Example

For low power applications, a single supply is usually available, which is usually somewhere between 1.8V to 3.6V. The design choices for the circuit shown in **Figure 4** depend on the output of the sensor and the application's requirements. One sensor which could correspond to this situation is the ADLX356B, which is an accelerometer used to measure the vibration of a machine, in a very noisy environment with many high frequency interferers. Example design key specifications for the circuit in **Figure 4** are listed in **Table 1**.

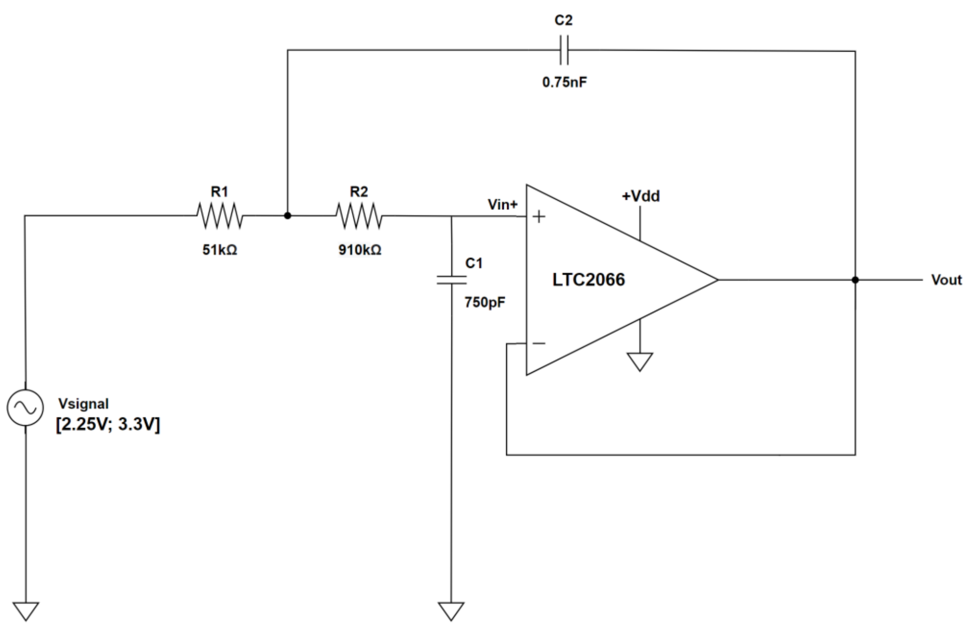

Figure 4: Sallen-Key low pass filter with unity gain.

Table 1. Design Key Specifications for circuit in Figure 4

Supplies V_{DD}/V_{SS}	Supply Current Max	Sensor Vsignal Min/Max Voltage	Cut-off frequency F_C	Stopband frequency Supplies +Vs/-Vs
+3.3V/0V	15µA	2.25V/3.3V	3kHz	25kHz

Design Description

The circuit in **Figure 4** is composed of the LTC2066, 10µA Supply current, low I_B, Zero-Drift Operational Amplifier which can be powered with a minimum 3.3V supply, V_{DD}. The LTC2066 comes in a 6-lead SC70 package. This low pass Sallen Key filter is used to filter the output of an accelerometer, to prevent aliasing in the band of interest.

The output of the filter is represented as V_{out}, while V_{signal}, represents the output of the accelerometer.

Design Considerations

1. The circuit in **Figure 4** is a Sallen-Key filter where no gain is implemented. A resistor network can be added to the op-amp feedback loop to add gain to the circuit (see Appendix 1). The quality factor Q, measures the performance, in terms of stability, is intrinsically linked to the gain. Keep the gain lower than 3 for best stability.

2. The quality factor Q can be important for Sallen-Key filters. In this design, Q is kept at $\sqrt{2}/2$, which is the standard value of a Butterworth second order filter. The Q factor can be adjusted to change the filter response. Higher Q (Chebyshev) gives faster roll off, by sacrificing phase linearity and magnitude flatness. Lower Q (Bessel) provides better phase linearity, by giving up rejection. Other parameters are important to describe a low-pass filter as shown in **Figure 5**.

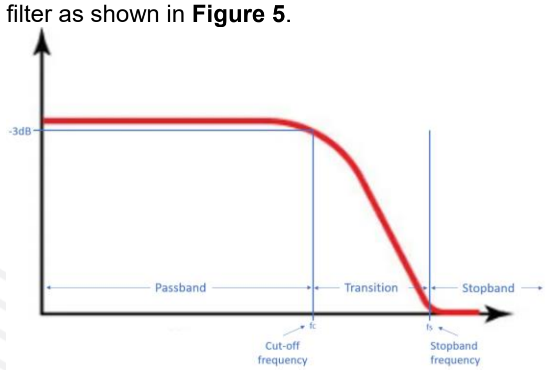

Figure 5: Usual behavior of a low-pass filter.

3. Unlike the ideal low-pass filter response, the Sallen Key filter will not be decreasing continuously past the cut-off frequency. After the cut-off frequency, the filter gain will decrease at 40dB/decade until a certain frequency (depending on the chosen amplifier) where the filter gain will start to increase at 20dB/decade until becoming constant. This behavior comes from the fact that Op-Amps are not ideal, they have finite gain bandwidth and an output impedance that is not zero. This finite gain bandwidth will cause the Op-Amp to run out of steam at frequencies that exceed its capability. When this happens, the Op-Amp looks like an output impedance connected to ground as the capacitors C_1 and C_2 will appear as a short circuit.
   Not only should the behavior in high frequency be monitored but it is also important to know at what frequency it starts to increase again as shown in **Figure 6** in green whereas the blue curve is the ideal behavior. This limitation is called the stop band limitation.

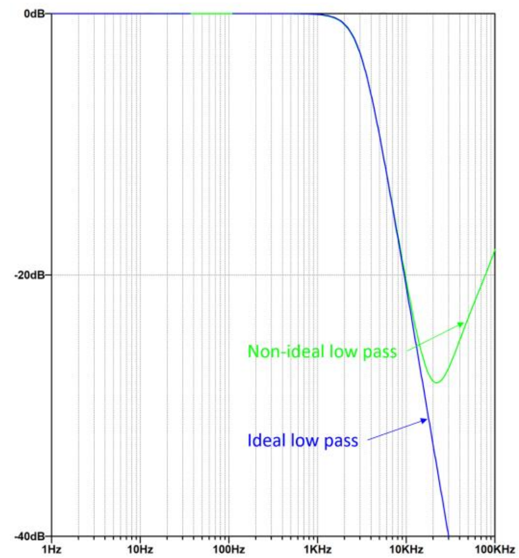

Figure 6: Stopband limitation.

4. In a Sallen Key filter, the chosen values for the resistors and capacitors are inversely proportional meaning that if you implement a larger resistor, you will have smaller capacitors.
   The use of large resistors and small capacitors is useful when the sensor is not great at driving the filter, such that it does not load the sensor. Finally, the Sallen-Key filter introduces an issue as its ability to reject signals decreases at high frequency as explained in design tip 3. Larger resistance reduces this issue.
   Smaller resistors and large capacitors have also some benefits. Small resistance will help the DC performance. Input bias current is flowing into the Op-Amp input, so the lower the resistor value, the lower the DC offset will be. This combination also allows lower noise. Finally, at higher frequency, small resistance will reduce the impact of parasitic capacitances on the design.
   For this low power design, it will be preferable to use large resistors, and small capacitors

5. When choosing passive component values (resistors and capacitors), the tolerances should be considered.
   For instance, when choosing the values R_1, R_2, C_1 and C_2, the tolerance of those components may alter the filter profile such as the cut-off frequency. In multichannel solutions, the component tolerance will also impact the matching between channels.

Design Procedure

1. *Setting the cutoff frequency for the filter:*

The sensor specifications are important to consider before proceeding to the design of the Sallen-Key filter. For this design, the relevant bandwidth for the accelerometer is 3kHz, so the Sallen-Key filter design will have a target cut-off frequency set to f_c=3kHz.

Note: The equation to measure the cut-off frequency from the low-pass Sallen Key transfer function is:

$$f_c=\frac{1}{2\pi\sqrt{R_1\times R_2\times C_1\times C_2}}$$

2. *Setting the Q of the filter:*

To realize a Sallen-Key filter and assess the values of the components, multiple ways are possible. In the case, where no gain is implemented, a common method, which doesn't limit the design, is to set the two resistors and two capacitors using ratios. Meaning, the resistors R_1 and R_2 are set such as:

$$R_1=m\times R=m\times R_2$$

Whereas the capacitors C_1 and C_2 are:

$$C_2=n\times C=n\times C_1$$

The first step is to choose the value of m and n. From design tips 2, the quality factor is $Q=\sqrt{2}/2\approx0.707$ but it is also possible to have a simplification of the expression of the Q factor coming from the transfer function as the gain, K, is equal to 1:

$$Q=\frac{\sqrt{m\times n}}{m+1+m\times n(1-K)}=\frac{\sqrt{m\times n}}{m+1}$$

As there are more unknown parameters than known parameters, it is typical to fix one ratio. A common value is n=10 which, will be used in this design procedure. With n=10, it is now possible to find the value of m.

$$Q=\frac{\sqrt{m\times n}}{m+1}$$

$$Q^2\times m^2+m(2\times Q^2-n)+Q^2=0$$

The full resolution of this second-order equation can be found in the appendix and the two solutions for m are:

$$m_1=\frac{-b-\sqrt{\Delta}}{2a}=\frac{-(2.Q^2-10)-\sqrt{\Delta}}{2.Q^2}=9-4\sqrt{5}$$

$$m_2=\frac{-b+\sqrt{\Delta}}{2a}=\frac{-(2.Q^2-10)+\sqrt{\Delta}}{2.Q^2}=9+4\sqrt{5}$$

The solutions m_1 and m_2 are the two values possible to get the quality factor equal to $\sqrt{2}/2$.

3. *Component values:*

From Design Considerations 4, the expectation was to use large resistors and small capacitors, within reason. Now that the ratios are known values, the next step is to select the value of R_1 to calculate the other component values. A value which could be considered as reasonably high is R_1=51kΩ, which is a standard resistor value. It is also high enough for the output impedance of the LTC2066, a low power Op-Amp to not be too large. From the previous step, two solutions for the resistor ratio were found. In the Sallen Key filter, for impedance reasons, it is better to have the low value of resistance at R_1 position, hence, the m_1 solution will be kept.
From the value of R_1 established previously and the chosen ratio, R_2 can be calculated.

$$R_1=m_1\times R=51k$$

$$R=R_2=\frac{R_1}{m_1}=\frac{51k}{9-4\sqrt{5}}=915k\Omega$$

The closest resistor standard value is 910kΩ.
Once the resistors values have been chosen and calculated, and thanks to the simplified expression of the cut-off frequency, the value of C can be calculated, and then, the values of both capacitors.

$$f_c=\frac{1}{2\pi\sqrt{R_1\times R_2\times C_1\times C_2}}$$

$$C=\frac{1}{2\pi\times f_c\times\sqrt{R_1\times R_2\times n}}$$

$$C=\frac{1}{2\pi\times 3kHz\times\sqrt{51k\times 910k\times 10}}=77pF$$

$$C=C_1=77pF$$

From the previous calculation, the closest standard capacitor value for C_1 is 75pF. Thanks to the ratio established previously, the value of C_2 can be calculated.

$$C_2=n\times C_1$$

$$C_2=10\times 75pF=0.75nF$$

The closest standard value is 0.75nF.

4. *Stop band limitation frequency:*

Considering the Design Considerations 6, in the design of the Sallen Key low-pass filters, the filter gain will ultimately start to increase again even though it is a low-pass filter. This behavior comes from the amplifier's output impedance interacting with the value of the resistance of the design, R_1 and R_2. The higher the value of those resistances, the further away the stop-band limitation will appear.

Design Simulations

The [Analog Filter Wizard Tool] provides filter recommendations and aids in the design of filters (low-pass, high-pass and band-pass) according to the specifications of the user. It will also help retrieve the expected performance of the chosen filter, such as the magnitude, phase, noise, the input impedance, the quiescent power consumption, and a recommendation for the circuit. It also gives critical analysis of the expected theoretical and practical performances of the filter.

The first step of using this tool is to refer to the specifications of the target low pass filter design. To have a second order filter with only one-stage, the Stopband specifications need to meet some requirements. According to the decibel value of the stopband, the interval of frequency to have a second order filter is different. In **Table 2**, you can find the frequency interval which will set a second order filter when choosing a cut-off frequency of 3kHz for different decibel stop band values.

Table 2: Behavior results interval

Stopband (dB)	Minimum frequency	Maximum frequency
20	10kHz	30kHz
25	13kHz	54kHz
30	17kHz	95kHz
40	30kHz	300kHz

From the design procedure, the characteristic of the passband is a gain of 0dB and a cut-off frequency at 3kHz (at -3B). The stopband specifications will be -30dB at 25kHz. Finally, it is possible to choose the filter response, meaning which type of filter you want to use (Butterworth Chebyshev, Bessel….). As explained in Design Considerations 2, and to be as close as possible to the theory above, a second order Butterworth will be chosen. From the first window, it is possible to read the number of stages from the specifications just entered as shown in **Figure 7**.

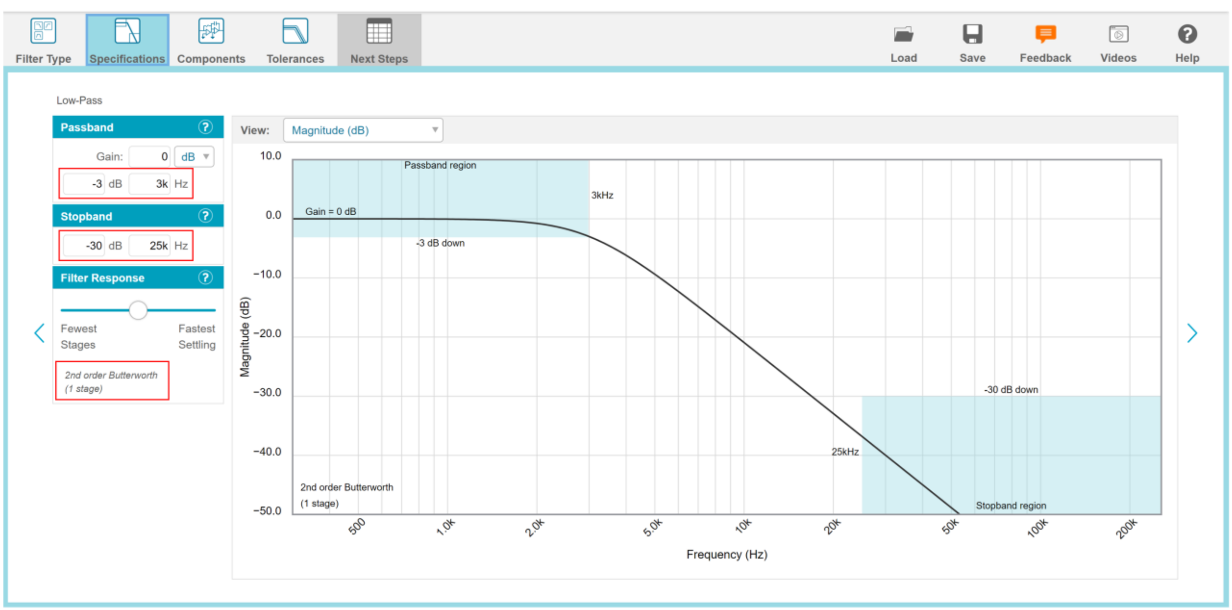

Figure 7: Analog Filter Wizard Tool, specifications window.

In the specifications window, a magnitude view can be retrieved. It is what the frequency output of the filter should look like in ideal conditions. Indeed, at high frequencies, the filter keeps rolling-off.
Once the specifications are entered, left click on the "*components*" button that is shown in **Figure 8**. In this window, it is possible to choose the values of the resistor and the capacitor but also choose the Op-Amp reference by clicking on the "*I want to choose*" from the category component. You can also choose the implementation of the filter as either Sallen Key or Multi Feedback To be as close as possible to the design procedure, the LTC2066 will be choose. The tool will also provide standard values for the resistors and the capacitors, which will be chosen, again, as close as possible to the one calculated previously.
Once all the components are set, the behavior of the filter can be monitored as on the **Figure 8**. Contrary to the **Figure 7**, the magnitude view is, this time, simulated with a true Op-Amp instead of an ideal one, which introduces the stop-band frequency at high frequencies. The summary of the behavior results is shown in **Table 3**.

Table 3. Design goal vs. Simulation AC analysis

Parameter	Design Goal	Simulation
C_1	75pF	75pF
C_2	0.75nF	0.75nF
R_1	51kΩ	52.7kΩ
R_2	910kΩ	946kΩ
Q	0,707	0.707

From the design procedure design goals and the Analog Filter Wizard result, it seems that the output is relatively similar in terms of component values.

Other results can be monitored to have a better understanding of the Sallen-Key low pass provided by the ADI tool such as the power or the RMS noise.

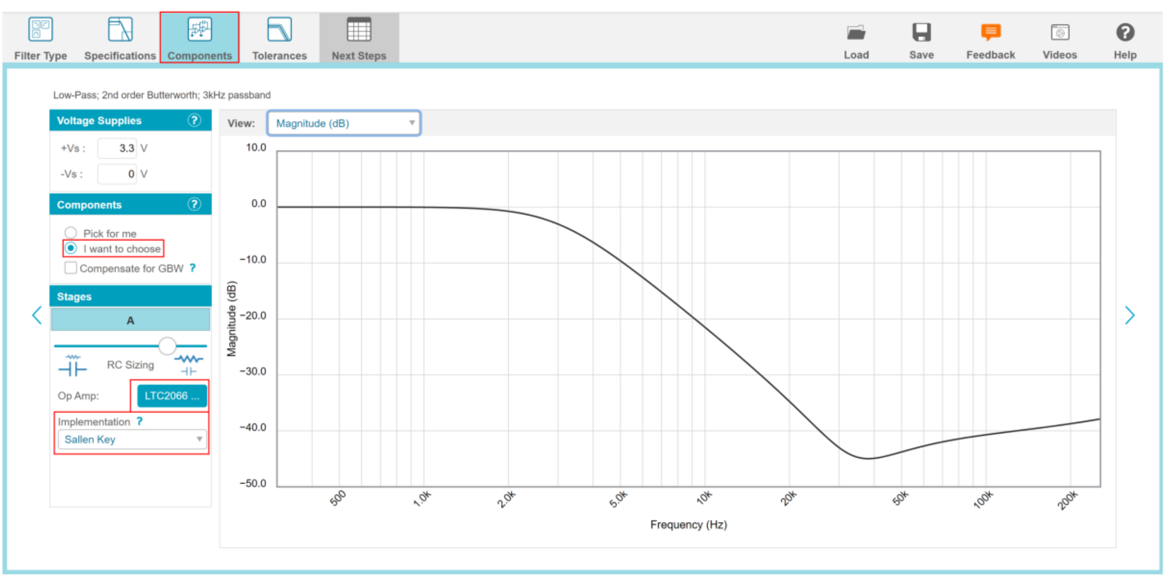

Figure 8. Analog Filter Wizard Tool: circuit result.

LTspice is also a great simulation tool to check against the design procedure calculations made above.
The LTspice schematic is shown in **Figure 9** below. The simulation done is a frequency response of the circuit and the result can be retrieved from the **Figure 10** with an input signal of 1V at 1kHz. The cursor from the **Figure 10** was placed at the -3dB point for the low pass filter. The **Table 4** shows a comparison of Design Goal vs. Simulation results.

Table 4. Design goal vs. Simulation AC analysis

Parameter	Design Goal	Simulation
Cutoff f_c	3kHz	2.982kHz
20log(gain)	0dB	-18µdB

Even if the cut-off frequency is quite close to the expectation, the span has different origins. First, the value of the resistors which were taken to be standard values but as close as possible to the values of the one calculated. This design shows a stopband of at least -20dB up to 80kHz. If this is not sufficient for the design, a wider bandwidth op-amp could be considered at the expense of higher power. Also, a simple R/C low pass filter following the Sallen-Key Filter could be considered to get more attenuation at higher frequencies.

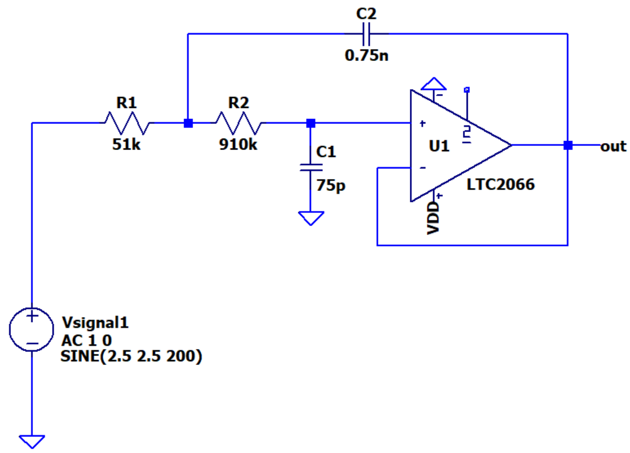

Figure 9: LTspice schematic.

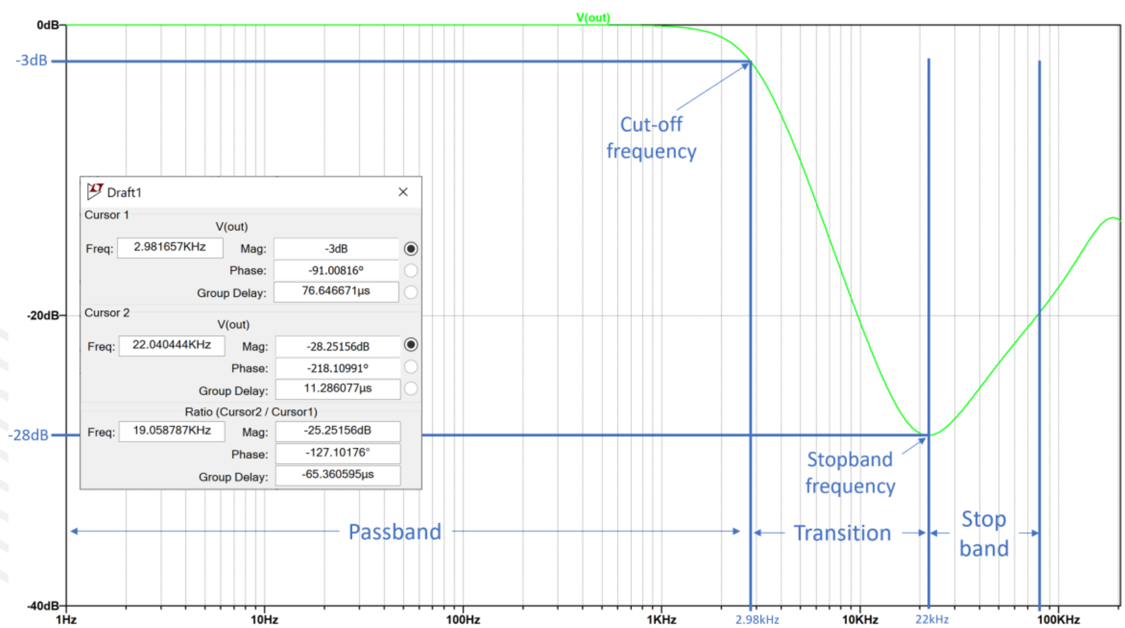

Figure 10. AC simulation results.

Design Devices

Table 5. Op-Amp

Part Number	Package size (SC70)	I_{bias} (A) max	V_{os} (V) max	0.1 to 10Hz Noise (uV p-p) typ	Vnoise (nV/rt-Hz) typ	Iq/Amp (A) typ	+Vs min/max (V)
LTC2066	2.40 mm x 2.20 mm	150pA	5µ	1.9	90	7.4µ	1.7/5.25

References

LTspice

LTspice® is a high-performance SPICE III simulator, schematic capture and waveform viewer with enhancements and models for easing the simulation of switching regulator, linear, and signal chain circuits.

Analog Filter Wizard

The Diamond Plot Tool is a web application that generates filter recommendations and aids in the design of low-pass, high-pass, or band-pass filters with the required specifications and with shorter lead times, for Analog Devices components.

Appendix

Appendix 1:

If a gain, K, is needed, the equation used will not be similar. The equation for the quality factor will be:

$$Q=\frac{\sqrt{R_1\times R_2\times C_1\times C_2}}{R_1\times C_1+R_2\times C_1+R_1\times C_2(1-K)}$$

In this situation, there is a link between the quality factor and the cut-off frequency. To answer the design specifications while setting the components values as ratios as previously, the design procedure should start by establishing the gain K and the resistance and capacitor ratio m and n to set Q. Then, the procedure should set the C value and finally, calculate the value of R according to the wished cut-off equation, f_c as:

$$f_c=\frac{1}{2\pi\times R\times C\times\sqrt{m\times n}}$$

And the gain K will be equal to:

$$K=1+\frac{R_4}{R_3}=1+\frac{m+1}{m\times n}$$

The gain can change the bandwidth of the Op-Amp, and this can affect the behavior of the passband and stopband. The output impedance of the amplifier is divided by the loop gain, therefore the output impedance will rise with increasing frequency. Small R_3 and R_4 resistances will also affect the response of the filter by shifting the stopband frequency towards the high frequency.

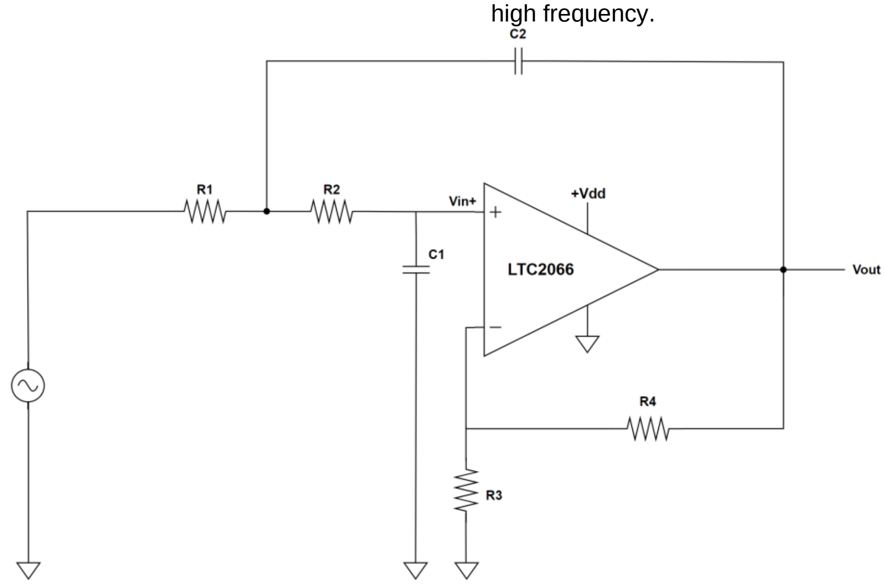

Figure 11. Sallen-Key low pass filter with gain.

Appendix 2:

The second order equation to resolve is under the format:

$$Q^2\times m^2+m(2\times Q^2-n)+Q^2=0$$

$$a\times m^2+m\times b+c=0$$

The delta of such equation is:

$$\Delta=b^2-4.a.c=(2\times Q^2-n)^2-4\times Q^2\times Q^2$$

$$\Delta=(2\times Q^2-n)^2-4\times Q^2\times Q^2$$

$$\Delta=(2\times(\frac{\sqrt{2}}{2})^2-10)^2-4\times(\frac{\sqrt{2}}{2})^2\times(\frac{\sqrt{2}}{2})^2$$

With n = 10 and $Q=\frac{\sqrt{2}}{2}$:

$$\Delta=80$$

As the delta is superior to zero, the two solutions are:

$$m=\frac{-b\pm\sqrt{\Delta}}{2a}=\frac{-(2.Q^2-10)\pm\sqrt{\Delta}}{2.Q^2}$$

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