SigmaStudio Filter Coefficient Calculations

This document shows how to calculate $B_0$, $B_1$, $B_2$, $A_1$, and $A_2$ for various IIR filter types in SigmaStudio.

$\sin = \text{sine}, \cos = \text{cosine}, \tan = \text{tangent}, \sinh = \text{hyperbolic sine}, \log = \text{base 10 logarithm}$

Parametric EQ Calculation

Values from Control:

- **boost** (boost of filter)
- **frequency** (center frequency)
- $Q$ (Q of filter; must be greater than or equal to 0.01)
- **gain** (linear gain applied to the signal)
- **Fs** (sample rate of project)

In the case that $\text{boost} = 0$,

- $\text{gainlinear} = 10^{(\text{gain} / 20)}$
- $B_0 = \text{gainlinear}$
- $B_1 = 0$
- $B_2 = 0$
- $A_1 = 0$
- $A_2 = 0$

If $\text{boost}$ is not 0,

Create 5 new variables: $a_0$, $\omega$, $\sin$, $\cos$, $\alpha$, $Ax$

- $Ax = 10^{(\text{boost} / 40)}$
- $\omega = 2 \times \pi \times \text{frequency} / \text{Fs}$
- $\sin = \sin(\omega)$
- $\cos = \cos(\omega)$
- $\alpha = \sin / (2 \times (Q))$

- $a_0 = 1 + (\alpha / Ax)$
- $A_1 = -(2 \times \cos) / a_0$
- $A_2 = (1 - (\alpha / Ax)) / a_0$
- $\text{gainlinear} = 10^{(\sin / 20)} / a_0$

- $B_0 = (1 + (\alpha \times Ax)) \times \text{gainlinear}$
- $B_1 = -(2 \times \cos) \times \text{gainlinear}$
- $B_2 = (1 - (\alpha \times Ax)) \times \text{gainlinear}$
**Tone Control**

Values from Control:

- **Freq_T** (treble cutoff frequency)
- **Boost_T** (treble boost)
- **Freq_B** (bass cutoff frequency)
- **Boost_B** (bass boost)
- **Fs** (sample rate of project)

\[
\begin{align*}
\text{Boost}_T &= 10^{(\text{Boost}_T / 20)} \\
\text{Boost}_B &= 10^{(\text{Boost}_B / 20)} \\
A_T &= \tan(\pi \times \frac{\text{Freq}_T}{\text{Fs}}) \\
A_B &= \tan(\pi \times \frac{\text{Freq}_B}{\text{Fs}}) \\
\text{Knum}_T &= 2 / (1 + (1 / \text{Boost}_T)) \\
\text{Kden}_T &= 2 / (1 + \text{Boost}_T) \\
\text{Knum}_B &= 2 / (1 + (1 / \text{Boost}_B)) \\
\text{Kden}_B &= 2 / (1 + \text{Boost}_B) \\
a_{10} &= A_T + \text{Kden}_T \\
b_{10} &= A_T + \text{Knum}_T \\
a_{11} &= A_T - \text{Kden}_T \\
b_{11} &= A_T - \text{Knum}_T \\
a_{20} &= (A_B \times \text{Kden}_B) + 1 \\
b_{20} &= (A_B \times \text{Knum}_B) - 1 \\
a_{21} &= (A_B \times \text{Kden}_B) - 1 \\
b_{21} &= (A_B \times \text{Knum}_B) + 1 \\
a_0 &= a_{10} \times a_{20} \\
A_1 &= ((a_{10} \times a_{21}) + (a_{11} \times a_{20})) / a_0 \\
A_2 &= a_{11} \times a_{21} / a_0 \\
\text{gainlinear} &= 10^{(\text{cell\_gain} / 20)} \\
B_0 &= (b_{10} \times b_{20}) / a_0 \times \text{gainlinear} \\
B_1 &= ((b_{10} \times b_{21}) + (b_{11} \times b_{20})) / a_0 \times \text{gainlinear} \\
B_2 &= (b_{11} \times b_{21}) / a_0 \times \text{gainlinear}
\end{align*}
\]

For double 1st Order Filter cells, simply use 2 Cascaded 1st Order Filters.
All Pass

Values from control:

**frequency** (center frequency)

Q (Q of filter)

**gain1** (linear gain)

fs (sample rate of project)

\[
\begin{align*}
\text{gain1} & = 10^{\left(\text{gain} / 20\right)} \\
\omega & = 2 \times \pi \times \text{frequency} / \text{fs} \\
s\sin & = \sin(\omega) \\
c\cos & = \cos(\omega) \\
\alpha & = s\sin / (2 \times Q) \\
\text{norm} & = 1 + \alpha \\
B0 & = \text{gain1} \times (1 - \alpha) / \text{norm} \\
B1 & = \text{gain1} \times (-2 \times c\cos) / \text{norm} \\
B2 & = \text{gain1} \times (1 + \alpha) / \text{norm} \\
A1 & = -2 \times c\cos / \text{norm} \\
A2 & = (1 - \alpha) / \text{norm}
\end{align*}
\]

Notch Filter

Values from control:

**frequency1** (center frequency)

Q (Q of filter)

g (gain)

fs (sample rate of project)

\[
\begin{align*}
\omega & = \text{frequency1} \times 2 \times \pi / \text{fs} \\
\delta\omega & = \omega / Q \\
b & = 1 / (1 + \tan(\delta\omega / 2)) \\
\text{gain} & = 10^{\left(\text{g} / 20\right)} \\
B0 & = \text{gain} \times b \\
B1 & = \text{gain} \times (-2 \times b \times \cos(\omega)) \\
B2 & = \text{gain} \times b \\
A1 & = -2 \times b \times \cos(\omega) \\
A2 & = (2 \times b - 1)
\end{align*}
\]
Chebyshev Low Pass

Values from control:

frequency (cutoff frequency; must be 1 Hz or greater)
gain (linear gain)
ripple (ripple of filter; must be 0.1 or greater)
Fs (sample rate of project)

\[
\begin{align*}
wp &= \frac{2 \cdot \pi \cdot \text{frequency}}{\text{Fs}} \\
\Omega_{\text{om}} &= \tan(wp / 2) \\
\epsilon_{\text{pass}} &= (10^{0.1 \cdot \text{ripple}} - 1)^{0.5} \\
\alpha &= (0.5) \cdot \log\left(\frac{1}{\epsilon_{\text{pass}}} + \frac{1}{(\epsilon_{\text{pass}}^2 + 1)}\right)^{0.5} \\
\Omega_{\theta} &= \Omega_{\text{om}} \cdot \sinh(\alpha) \\
\theta &= \frac{\pi}{4} \cdot 3 \\
\Omega_{\text{m}} &= \Omega_{\text{om}} \cdot \sin(\theta) \\
H_0 &= \frac{1}{\left(1 + \frac{1}{\epsilon_{\text{pass}}^2}\right)^{0.5}} \\
G &= \frac{\Omega_{\theta}^2 + \Omega_{\text{m}}^2}{\text{Den}} \\
A_1 &= \frac{2 \cdot \left(\Omega_{\theta}^2 + \Omega_{\text{m}}^2 - 1\right)}{\text{Den}} \\
A_2 &= \frac{1 + \left(2 \cdot \Omega_{\theta} \cdot \cos(\theta)\right) + \Omega_{\theta}^2 + \Omega_{\text{m}}^2}{\text{Den}} \\
B_0 &= H_0 \cdot G \cdot 10^{\left(gain / 20\right)} \\
B_1 &= B_0 \cdot 2 \\
B_2 &= B_0
\end{align*}
\]
ChebyshevHighPass

Values from control:

**frequency** (cutoff frequency; must be 1 Hz or greater)
**gain** (linear gain)
**ripple** (ripple of filter; must be 0.1 or greater)
**Fs** (sample rate of project)

\[
\begin{align*}
wp &= \left(2 \times \pi \times \text{frequency}\right) / \text{Fs} \\
\text{Omegap} &= 1 / \tan(wp / 2) \\
\text{epass} &= \left(10^{0.1 \times \text{ripple}} - 1\right)^{0.5} \\
\text{alpha} &= \left(0.5 \times \log\left(1 / \text{epass} + \left(1 / \left(\text{epass}^2\right) + 1\right)\right)\right)^{0.5} \\
\text{Omega}0 &= \text{Omegap} \times \sinh(\text{alpha}) \\
\text{theta} &= \left(\pi / 4\right) \times 3 \\
\text{Omega}1 &= \text{Omegap} \times \sin(\text{theta}) \\
\text{H0} &= \left(1 / \left(1 + (\text{epass}^2)\right)\right)^{0.5} \\
\text{Den} &= 1 - 2 \times \text{Omega}0 \times \cos(\text{theta}) + \left(\text{Omega}0^2\right) + \left(\text{Omega}1^2\right) \\
\text{G} &= \left(\left(\text{Omega}0^2\right) + \left(\text{Omega}1^2\right)\right) / \text{Den} \\
\text{A1} &= \left(-2 \times \left(\left(\text{Omega}0^2\right) + \left(\text{Omega}1^2\right) - 1\right)\right) / \text{Den} \\
\text{A2} &= \left(1 + 2 \times \text{Omega}0 \times \cos(\text{theta}) + \left(\text{Omega}0^2\right) + \left(\text{Omega}1^2\right)\right) / \text{Den} \\
\text{B0} &= \text{H0} \times \text{G} \times \left(10^{\text{gain} / 20}\right) \\
\text{B1} &= -\text{B0} \times 2 \\
\text{B2} &= \text{B0}
\end{align*}
\]
Linkwitz-Riley – 12 dB/oct = 2 cascaded 1st order butterworths (2 biquads)

**frequency** (the cutoff frequency)
**First Order type** (the type of filter, can be lowpass, highpass, or allpass)
**g** (the linear gain)
**fs** (the sample rate of the project)

1st Order Butterworth

\[
\text{gain } = 10^{\frac{g}{20}}
\]

\[
\omega, \text{ sn, cs, alpha, a0}
\]
\[
\omega = 2 \times \pi \times \text{frequency} / \text{fs}
\]
\[
\text{sn} = \sin(\omega)
\]
\[
\text{cs} = \cos(\omega)
\]
\[
a0 = \text{sn} + \text{cs} + 1;
\]
For a First Order lowpass...

\[
A1 = \frac{(\text{sn} - \text{cs} - 1)}{a0}
\]
\[
B0 = \text{gain} \times \text{sn} / a0
\]
\[
B1 = \text{gain} \times \text{sn} / a0
\]

For a First Order highpass...

\[
A1 = \frac{(\text{sn} - \text{cs} - 1)}{a0}
\]
\[
B0 = \text{gain} \times (1 + \text{cs}) / a0
\]
\[
B1 = - \text{gain} \times (1 + \text{cs}) / a0
\]

For a First Order allpass...

\[
A1 = (2.7 \times 2 \times \pi \times \text{frequency} / \text{fs})
\]
\[
B0 = - \text{gain} \times A1
\]
\[
B1 = \text{gain}
\]
Linkwitz-Riley – 24 dB/oct = 2 cascaded 2nd order butterworths (2 biquads)

frequency (the cutoff frequency)
gain (the linear gain)
Fs (the sample rate of the project)

2nd Order LOWPASS
omega, sn, cs, alpha, a0;
omega = 2 * PI * frequency / Fs
sn = Sin(omega)
cs = Cos(omega)

alpha = sn / (2 * (1 / (2)^0.5))
a0 = 1 + alpha
A1 = -( 2 * cs) / a0
A2 = (1 - alpha) / a0
B1 = (1 - cs) / a0 * (10^(gain / 20))

B0 = B1 / 2
B2 = B0

2nd Order HIGHPASS
omega = 2 * PI * frequency / Fs
sn = Sin(omega)
cs = Cos(omega)

alpha = sn / (2 * (1 / (2)^0.5))
a0 = 1 + alpha
A1 = -( 2 * cs) / a0
A2 = (1 - alpha) / a0
B1 = -(1 + cs) / a0 * (10^(gain / 20))

B0 = - B1 / 2
B2 = B0
Linkwitz-Riley – 36 dB/oct = 2 cascaded 3rd order butterworths
3rd order butterworth is implemented by cascading a “HigherOrder” + 1st order
1st Filter: [HigherOrder] orderindex = 3, i = 0
2nd Filter: 1st Order Butterworth
3rd Filter: [HigherOrder] orderindex = 3, i = 0
4th Filter: 1st order Butterworth

frequency (the cutoff frequency)
gain (the linear gain)
Fs (the sample rate of the project)
orderindex (described above… changes based on type)
i (described above)

LOW PASS HIGHER ORDER
omega, sn, cs, alpha, aθ, orderangle
omega = 2 * PI * frequency / Fs
sn = Sin(omega)
cs = Cos(omega)

orderangle = (PI / orderindex) * (i + 0.5)
alpha = sn / (2 * (1 / (2 * Sin(orderangle))))
aθ = 1 + alpha
A1 = -( 2 * cs) / aθ
A2 = (1 - alpha) / aθ
B1 = (1 - cs) / aθ * (10^(gain / 20))
B0 = B1 / 2
B2 = B0

HIGH PASS HIGHER ORDER
omega = 2 * PI * frequency / Fs
sn = Sin(omega)
cs = Cos(omega)

orderangle = (PI / orderindex) * (i + 0.5)
alpha = sn / (2 * (1 / (2 * Sin(orderangle))))
aθ = 1 + alpha
A1 = -( 2 * cs) / aθ
A2 = (1 - alpha) / aθ
B1 = -( 1 + cs) / aθ * (10^(gain / 20))
B0 = - B1 / 2
B2 = B0

Linkwitz-Riley – 48 dB/oct = 2 cascaded 4th order butterworths
4th order butterworth is implemented by cascading 2 2nd “Higher Order” using
equations shown above.
1st Filter: orderindex = 4, i = 0
2nd Filter: orderindex = 4, i = 1
3rd Filter: orderindex = 4, i = 0
4th Filter: orderindex = 4, i = 1
Butterworth 12 dB/oct = One 2nd order butterworth

Butterworth 18 dB/oct = One Higher order butterworth + One 1st Order
Filt 1: [HigherOrder]: orderindex = 3, i = 0
Filt 2: 1st Order butterworth

Butterworth 24 dB/oct = 2 Higher order butterworths
Filt 1: orderindex = 4, i = 0
Filt 2: orderindex = 4, i = 1

Bessel 12 dB/oct = One 2nd order Bessel

Bessel 18 dB/oct = one 2nd order Bessel + One 1st order Butterworth

Bessel 24 dB/oct = Two 2nd order Bessel